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HEAT TRANSFER TO CYLINDERS IN HELIUM AND HELIUM–AIR MIXTURES*

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Abstract—Measurements have been made of the heat transfer from platinum wires in the Reynolds number range 0·03–10 in helium and helium–air mixtures. Although no theory for such heat transfer exists, it is shown that a heuristic calculation procedure provides estimates for use in hot wire anemometry.

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NOMENCLATURE

c, concentration of helium, mass fraction;
d, diameter of wire;
E, voltage drop across the wire;
h, local heat-transfer coefficient;
k_m, thermal conductivity of the gas evaluated at T_m;
L, length of wire;
n, constant;
Nu, Nusselt number hd/k;
Nu_c, continuum value of Nusselt number;
Pr, Prandtl number;
R, wire resistance;
Re, Reynolds number, $U_\infty d/\nu_m$;
T_m, mean film temperature, $(T_w + T_\infty)/2$;
T_w, wire temperature;
T_∞, free stream temperature;
U_∞, free stream velocity.

Greek symbols

α, thermal accommodation coefficient;
β, slip parameter, cf. equation (1);
γ, ratio of specific heat;
λ, mean free path;
τ, over-heat-ratio;
ν_m, kinematic viscosity, evaluated at T_m;
χ, mole fraction of helium.

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INTRODUCTION

THE HOT wire anemometer in connection with modern data processing techniques provides a convenient device for measuring simultaneously the velocity, temperature and concentration of one species of a binary mixture in a turbulent flow. Such extended uses of the hot wire technique were suggested long ago by Corrsin [1]. In connection with its use in helium and air mixtures Aihara *et al.* [2] showed that thermal slip effects associated with the poor thermal accommodation of helium on common hot wire materials, e.g. tungsten, prevent use of a simple calibration scheme. Their experimental results for the heat loss from heated wires in pure gases were found to be in good agreement with the theory of Kassoy [3] which is self-consistent for pure species, e.g. in pure helium and pure air, but which requires an auxiliary calculation of a mixture slip parameter for binary mixtures. Further measurements and comments on this auxiliary calculation are given in [4].

The work in [2–4] is related to a Reynolds number range of 10^{-1} or less, i.e. a range which is one or more decades below that customarily of interest in hot wire anemometry. The present paper provides experimental data on the heat loss from cylinders in helium and helium–air mixtures in the Reynolds number range approximately 3×10^{-2}–10.

DESCRIPTION OF MEASUREMENTS

In the interest of brevity we do not give the details of the measuring techniques; they are essentially as given in [2] and involve large aspect ratio wires with guard heaters on the wire supports. Three different jets are used to establish the test flows over the entire Reynolds number range. The wires are platinum of 8×10^{-4} in. dia.

Most tests were performed at a wire overheat of 70°C so that the parameter indicative of the wire temperature in [3], namely $\tau \equiv (T_w - T_\infty)/T_\infty \simeq 0{\cdot}17$. However, some tests were performed with higher τ so that data are identified by concentration of helium in mass fraction thereof, c; and by τ.

From the voltage drop across the wire, the wire geometry and the wire resistance, the heat loss in a given flow condition is determined and is subsequently reduced to a Nusselt number,

$$Nu = \frac{E^2}{\pi LR(T_w - T_\infty)k_m}$$

where the subscript m denotes evaluation at the arithmetic mean of T_∞ and T_w. The variation of the Nusselt number with Reynolds number $Re = U_\infty d/\nu_m$ for a given concentration is the primary experimental data presented here.

To give an indication of the overall accuracy of the measurements there are shown in Fig. 1 the results for pure air compared with the correlation of Collis and Williams [5] which is based on their highly accurate measurements. Over the temperature range involved here the Nusselt–Reynolds number predictions of [5] are essentially independent of τ. It is seen from Fig. 1 that the present data are in excellent agreement with [5].

In the data reduction process certain properties of the gases and gas mixtures are required. The physical properties of the pure gases are taken from [6–8]. The binary mixture rules for viscosity and thermal conductivity are taken from Wilke [9] and, Lindsay and Bromley [10] respectively.

EXISTING THEORY

Before discussing the experimental results it

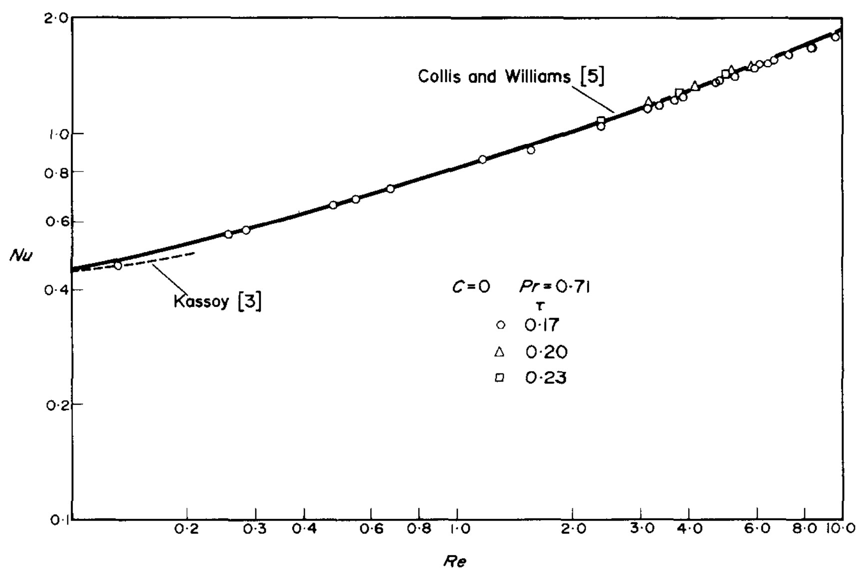

FIG. 1. Variation of Nusselt number with Reynolds number: $c = 0$.

is instructive to consider the available theory applicable to the data. In the intermediate Reynolds number range of interest here, there is no adequate theory even for strictly continuum flows. Thermal slip effects due to the poor thermal accommodation of helium on platinum present an additional complication. The only relevant available calculation appears to be that due to Sauer and Drake [11] which is based on a modified Oseen approximation, i.e. the acceleration term in the equation of momentum conservation in the peripheral direction of polar coordinates is replaced in an ad hoc manner by constant velocity (U_∞/n) times the radial gradient of the peripheral velocity component where n is to be determined either by comparison with experiment or by some *a posteriori* considerations. Variations of fluid properties are neglected so that the heat transfer in the form of a Nusselt number depends only on Reynolds number and on a slip parameter.

In comparing the theory of [11] with experimental results, two difficulties arise; for pure species the only ambiguity with respect to β, the slip parameter defined as

$$\beta \equiv \frac{2(2-\alpha)}{\alpha}\left(\frac{\gamma}{\gamma+1}\right)\left(\frac{\lambda}{d}\right)\frac{1}{Pr} \tag{1}$$

relates to the selection of the thermal accommodation coefficient, α, with its sensitivity to precise surface conditions. For mixtures of helium and air as has been pointed out in [2] and [4] there appears to be the additional complication that there exists no method of computing the effective α. Consequently it seems appropriate to consider β as one parameter dependent on concentrations to be determined from experiment.

The second difficulty concerns the selection of n. Sauer and Drake considered n to be a universal constant and to be selected so as to bring theory and experiment into agreement. For the case of pure air for which $\beta = 0$ in our experiments they found $n = 3$ by comparing with McAdams' empirical correlation. However, a more careful examination of the behavior of n which we have carried out in connection with the present experiments shows that it depends on both β and Re.

It thus appears more constructive to consider an alternative means for predicting heat-transfer behavior, that suggested by the analysis of Aihara *et al.* [2]. Consider as a definition of β

$$\frac{1}{Nu} - \frac{1}{Nu_c} = \frac{\beta}{2} \tag{2}$$

and the suggested heuristic "mixture rule" for helium–air mixtures,

$$\beta = \chi\beta_{He} \tag{3}$$

where χ is the mole fraction of helium and Nu_c is the continuum Nusselt number, given by a modified Collis and Williams correlation

$$Nu_c = \left(\frac{T_m}{T_\infty}\right)^{0\cdot17}\left[0\cdot24\left(\frac{Pr}{0\cdot71}\right)^{0\cdot2} + 0\cdot56\left(\frac{Pr}{0\cdot71}\right)^{0\cdot33} Re^{0\cdot45}\right]. \tag{4}$$

The dependence of the heat transfer on the Prandtl number given in equation (4) is obtained from Kramers [12]. As can be noted from equation (4), one would recover the Collis and Williams correlation for the case of air with $Pr = 0\cdot71$. Equation (2) used in conjunction with equations (3) and (4) allows one to estimate the Nusselt number, Nu for specified c, Re and Pr, provided that β for pure helium is known or estimated.

EXPERIMENTAL RESULTS

Shown in Figs. 2–6 are the experimental obtained distributions of Nusselt number with Reynolds number for some of the various helium concentrations. We also show the heat transfer distributions for air ($c = 0$) and for the particular mixture ($c \neq 0$) but without slip ($\beta = 0$). Since the lower limit for the validity of Collis and Williams empirical curve from equation (4) is at about $Re = 0\cdot02$, the theory of Kassoy [3] which is valid up to $Re \simeq 0\cdot1$ provides an excellent mean to check the accuracy of the

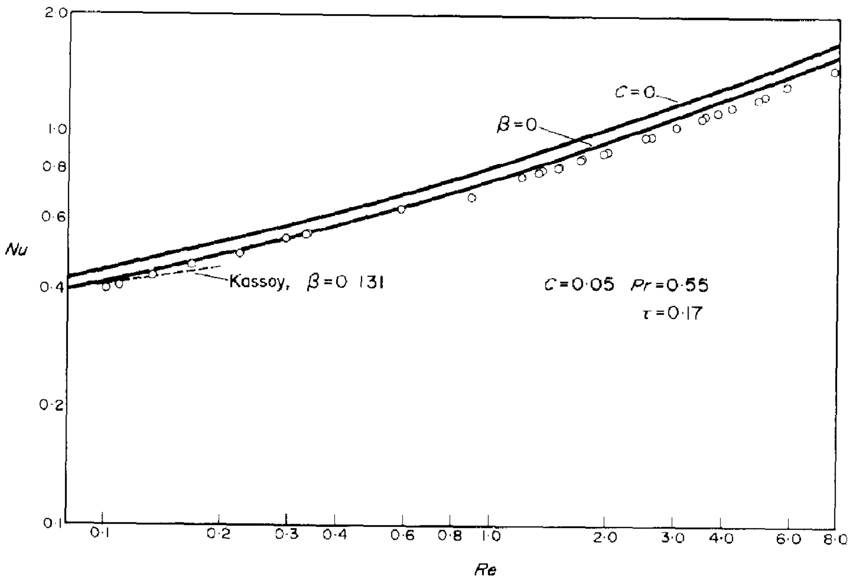

FIG. 2. Variation of Nusselt number with Reynolds number: $c = 0{\cdot}05$.

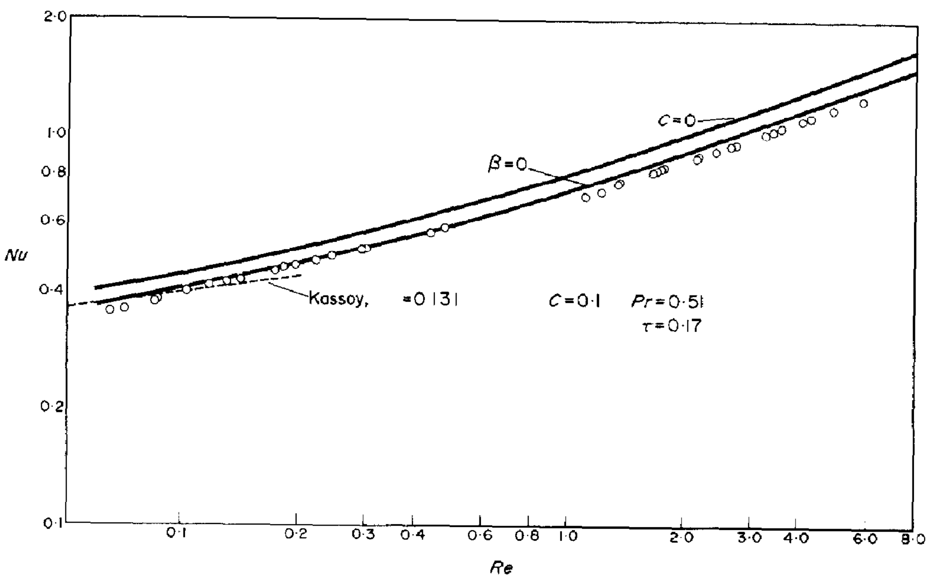

FIG. 3. Variation of Nusselt number with Reynolds number: $c = 0{\cdot}1$.

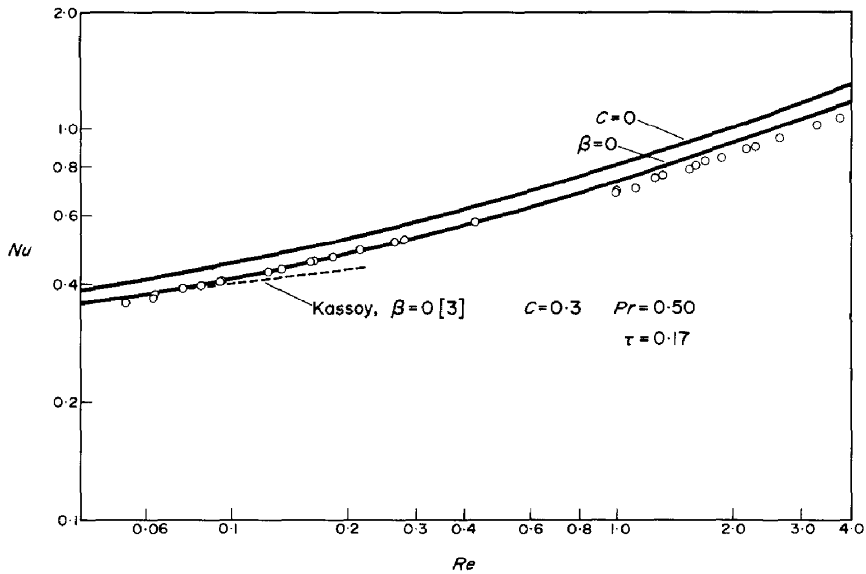

FIG. 4. Variation of Nusselt number with Reynolds number: $c = 0{\cdot}3$.

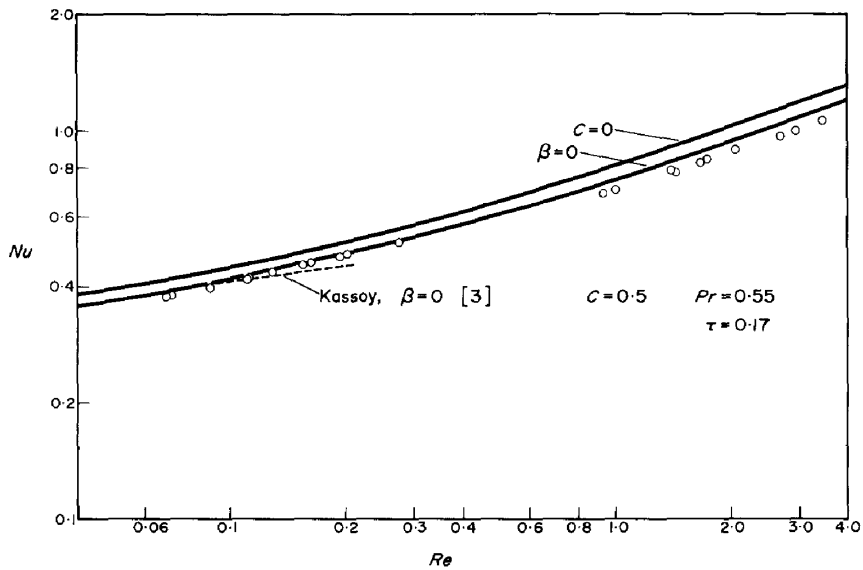

FIG. 5. Variation of Nusselt number with Reynolds number: $c = 0{\cdot}5$.

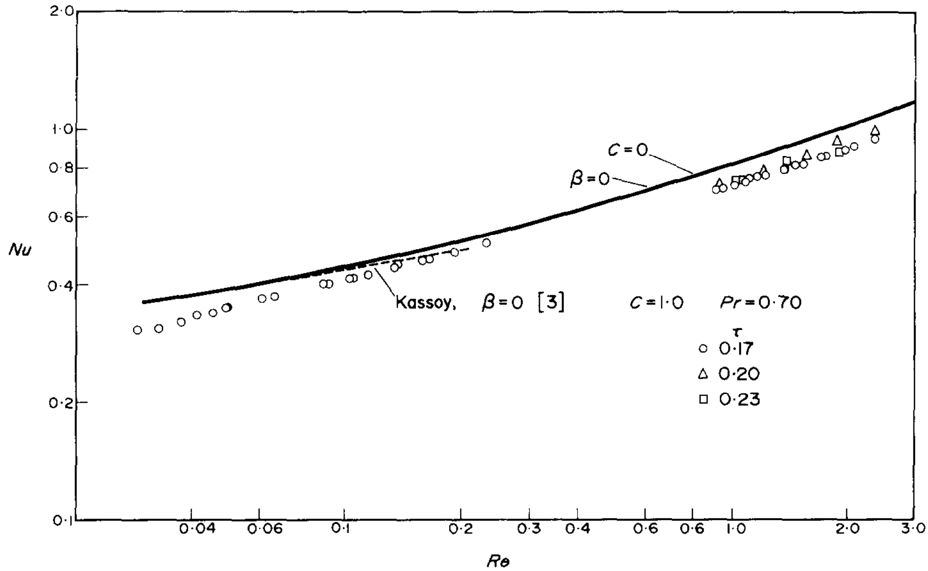

FIG. 6. Variation of Nusselt number with Reynolds number: $c = 1{\cdot}0$

$\beta = 0$ curve. We note that the part of the difference between the heat loss in pure air and in helium–air mixtures is accounted for by the change in Prandtl number but there remains discernible a slip effect (cf. in particular Figs. 5 and 6). At the lower Reynolds number this effect, while present, is not great enough to permit direct estimates of the accuracy of equations (2)–(4).

From the first set of the experimental data, i.e. those corresponding to the higher Reynolds number, we can by means of equations (2)–(4), calculate β as a function of concentration of helium. As shown in Fig. 7, β inferred from the experimental data is in rough agreement with, but generally falls below the heuristic "mixture rule" of equation (3) as has also been observed in [4]. The value of β_{He} is selected to be the arithmetic mean of the inferred β at $c = 1{\cdot}0$ and is 0·274 from which the thermal accommodation coefficient α can then be calculated from equation (1). The corresponding α is found to be 0·11 which falls within the range of the experimental values of α [13].

We thus conclude that equations (2)–(4) pro-

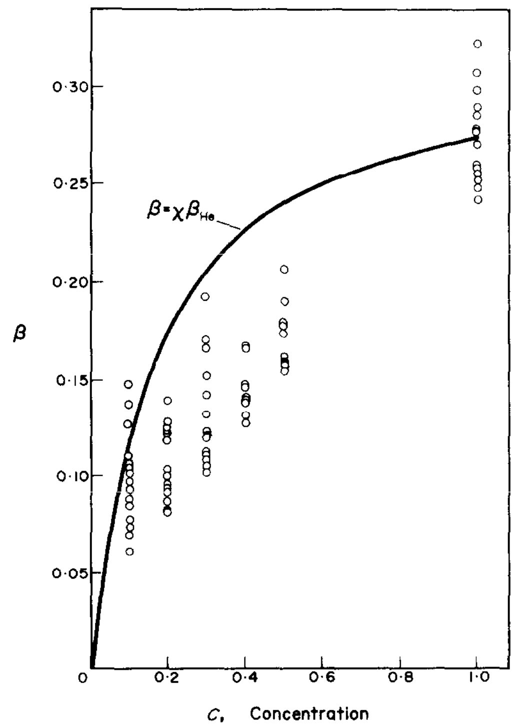

FIG. 7. Inferred slip parameter β at various concentrations of helium.

vide a useful recipe for estimating heat loss under the test conditions of the present experiments. In addition, of course the heat loss data should be significant in connection with the development of an adequate theory for this class of flows.

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TRANSFERT THERMIQUE À PARTIR DE CYLINDRES DANS DES MÉLANGES HÉLIUM ET HÉLIUM-AIR

Résumé—On a effectué des mesures de transfert thermique à partir de fils de platine dans un domaine de nombre de Reynolds compris entre 0·03 et 10 pour des mélanges hélium et hélium-air. Bien qu'il n'existe aucune théorie concernant un tel transfert thermique on montre qu'un procédé de calcul heuristique fournit des estimations pour l'application de l'anémométrie à fil chaud.

WÄRMEÜBERTRAGUNG AN ZYLINDER IN HELIUM UND HELIUM-LUFT-GEMISCHEN

Zusammenfassung—Es wurden Messungen durchgeführt, wobei die Wärmeübertragung von Platindrähten in Helium und Helium-Luft-Gemischen im Bereich der Reynoldszahlen 0,03 bis 10 gemessen wurde. Obwohl keine Theorie für solche Wärmeübertragungsvorgänge existiert, wird gezeigt, dass ein wegweisendes Berechnungsverfahren zur Verfügung steht, um Abschätzungen zum Gebrauch in der Hitzdraht-Anemometrie durchführen zu können.

ПЕРЕНОС ТЕПЛА К ЦИЛИНДРАМ В ГЕЛИИ И СМЕСИ ГЕЛИЙ-ВОЗДУХ

Аннотация—Проводились измерения переноса тепла от платиновых проволок в гелии или воздушно-гелиевых смесях при числах Рейнольдса от 0,03 до 10. Хотя для такого переноса тепла не существует какой-либо теории, показано, что эвристический расчёт даёт оценки, которые можно использовать в анемометрии на основе нагретой нити.